Reinvestigation of Buckling of Shells of Revolution by a Refined Finite Element

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Introduction

ASYMMETRIC buckling of shells of revolution using different numerical methods such as finite element,[1] integration,[2] and finite difference techniques,[3] has received considerable attention in the past few years. It was pointed out in Ref. 2 that the results of Navaratna et al.[1] for cylindrical shells of variable thickness under axial compression were in serious disagreement with the results obtained by the integration method. Further, the axial tensile buckling load of a truncated hemisphere using the finite element method[1] did not agree with the results of Yao,[4] Wu and Cheng,[5] and Radhamohan and Prasad.[6] These observations prompted this reinvestigation of the buckling analysis of shells of revolution using a more refined finite element. The numerical results based on the present formulation will be limited to clarify the discrepancies existing in the work of Navaratna et al.[1] In what follows, the salient features of the formulation are summarized.

Element Properties

A geometrically exact element for a shell of revolution is developed using the following displacement distributions

$$\begin{aligned} u &= (\alpha_1+\alpha_2 s+\alpha_3 s^2+\alpha_4 s^3)\cos j\theta \\ v &= (\alpha_5+\alpha_6 s+\alpha_7 s^2+\alpha_8 s^3)\sin j\theta \\ w &= (\alpha_9+\alpha_{10} s+\alpha_{11} s^2+\alpha_{12} s^3)\cos j\theta \end{aligned} \qquad (1)$$

where u is the meridional displacement, v is the circumferential displacement, w is the normal displacement, s is the meridional coordinate, θ is the circumferential coordinate and j is the circumferential wave number.

The generalized displacements $\alpha_1 \ldots \alpha_{12}$ are determined in terms of the nodal parameters u, $\partial u/\partial s$, v, $\partial v/\partial s$, w, β (where β is the meridional rotation) at the ends of the element.

Using Sanders' nonlinear strain displacement relations,[7] and following the general procedure of Ref. 1, the stiffness and geometric stiffness matrices are derived.

No approximation on the thickness variation and the radii of curvature is made as numerical integration is used to derive the element matrices.[8] It is noted that for the prebuckling stress analysis, linear strain displacement relations are used with $j = 0$ in Eq. (1) (axisymmetric analysis) in deriving the stiffness matrix.

Table 1 Buckling load of an axially compressed cylinder of constant thickness

	Shell geometry: $L/R = 1.75$; $R/t = 800$ Dimensionless buckling load λ^a		
Number of elements	Present work	Navaratna et al.	Hoff and Soong
10^b	$0.5505(1)^d$		
20^b	0.5142(1)		
20^c	0.5004(1)	0.5037(2)	0.505(3)

^{a} $\lambda = \sigma_{cr}/\sigma_{CL}$; $\sigma_{CL} = E/[3(1-\nu^2)]^{1/2}(t/R)$.
^{b} Uniform mesh.
^{c} Graded mesh with very fine mesh near the ends.
^{d} Numbers in brackets are circumferential waves of buckling.

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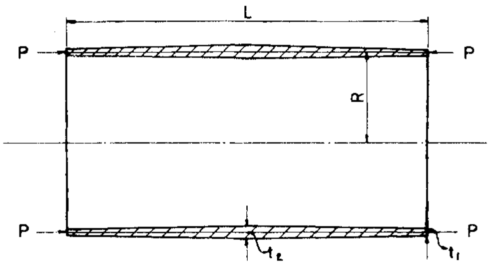

Fig. 1 Cylindrical shell of variable thickness with longitudinal compression.

Numerical Examples

The first example considered is buckling of a cylindrical shell of radius R, of length L, and of constant thickness t subjected to a longitudinal compression P (see Fig. 1 with $t_1 = t_2 = t$). For the prebuckling analysis, the normal displacement w is freely allowed. In the buckling analysis, the boundary conditions are $u = w = 0$ at both ends of the cylinder. Table 1 presents the nondimensional buckling load obtained with 10 elements uniform mesh and 20 elements uniform and graded meshes. For comparison the results of Navaratna et al.[1] and Hoff and Soong[9] are also included. Simmonds and Danielson[10] showed that for the aforementined boundary conditions, cylinders always buckle with wave number of unity. This fact is also observed by the present finite element results. On the contrary, Navaratna et al.[1] and Hoff and Soong[9] obtained higher wave numbers.

The second example is buckling of a cylindrical shell of variable thickness subjected to a longitudinal compression P as shown in Fig. 1. The boundary conditions used are the same as that of the first example. Table 2 presents the nondimensional buckling load of the cylinder by the present finite element solution by Navaratna et al.[1] and by Radhamohan.[2] The present finite element results are in very good agreement with the results of Radhamohan.[2]

Next example considered is that of a truncated hemisphere subjected to axial tension as shown in Fig. 2. The boundary conditions for the prebuckling analysis are the following: 1) $u = w = \beta = 0$, at the bottom edge of the shell, and 2) $\beta = 0$, $\delta = 1$, and radial displacement is zero, at the top edge of the shell. The boundary conditions used for the buckling analysis are $u = v = w = \beta = 0$ at top and bottom edges of the shell. This problem is solved by the present finite element with 10 and 20 elements using graded meshes and the nondimensional buckling load obtained is presented in Table 3 along with the results of Refs. 1, 4–6. It is observed from the table that the present finite element solution agrees very well with the results of Yao, Wu

Table 2 Buckling load of an axially compressed cylinder with linearly varying thickness

	Shell geometry: $L/R = 1.6$; $R/t_{av} = 100$ $t_1 = 0.005$ in.; $t_2 = 0.015$ in. Dimensionless buckling load λ^a		
Circumferential waves j	Present^b work	Navaratna et al.	Radhamohan
1	0.1596		0.1597
2	0.1609	0.5510	0.1609

^{a} $\lambda = \sigma_{cr}/\sigma_{CL}$; $\sigma_{CL} = E/[3(1-\nu^2)]^{1/2} t_{av}/R$.
^{b} Number of elements used is 20 with a very fine mesh near the ends.

Table 3 Buckling load for a truncated hemisphere subjected to a unit displacement on the top edge (See Fig. 2)

Circumferential wave number j	Shell geometry: $R/t = 480.0$; $\alpha = 23° 30'$ — Dimensionless buckling load $P/Et * 10^4$ — Present work[a] 10 Elements	Present work[a] 20 Elements	Yao	Wu and Cheng	Radhamohan and Prasad	Navaratna et al.
38	—	13.074			12.97	7.00
39	13.043	13.024			12.93	6.88
40	13.028	13.009	13.4903	13.0567	13.16	6.93
41	13.046	13.028	—	—	—	—

[a] Very fine mesh is used near the ends.

Table 4 Buckling load for a truncated sphere subject to a unit load on the top edge

Circumferential at wave number j	Shell geometry: $R/t = 480.0$; $\alpha = 23° 30'$ — Dimensionless buckling load $P/Et * 10^4$ — Number of elements 10[a]	Number of elements 20[a]
38	11.888	12.050
39	11.833	11.995
40	11.808	11.972
41	11.813	11.980

[a] Very fine mesh is used near the ends.

and Cheng, and Radhamohan and Prasad. However, the results of Navaratna et al. are in considerable disagreement with all other works.

The last example considered is that of truncated hemisphere of Fig. 2 subjected to axial tensile load. The boundary conditions for the prebuckling analysis are the following: 1) $u = w = \beta = 0$, at the bottom edge of the shell, and 2) $\beta = 0$, at the top edge of the shell. The boundary conditions used for the buckling analysis are $u = v = w = \beta = 0$, at the top and bottom edges of the shell. The problem is solved with 10 and 20 elements using graded meshes and the nondimensional buckling load obtained is presented in Table 4.

Conclusions

The numerical results obtained demonstrate that the present refined finite element yields accurate buckling loads for various shell geometries and agrees very well with the results of other numerical methods except that of Ref. 1.

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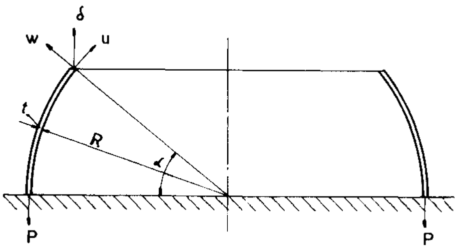

Fig. 2 Truncated hemispherical shell with axial tension.

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Solution for Inplane and Bending Fields of Tapered Anisotropic Conical Shells

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Introduction

RECENTLY Padovan has developed exact solutions for anisotropic laminated cylindrical shells,[1–3] plate strips,[4] circular plates[5,6] and half-space problems.[7,8] Outside of these configurations little is known about the effects of curvilinear material anisotropy. With this in mind, the present Note develops an exact solution for the static mechanical loading of anisotropic tapered conical shells with arbitrary edge conditions. In particular the type of taper treated is linearly dependent on the usual meridional variable used for conical shells.[9]

To illustrate the solution procedure, Donnell type cone equations[9] are used. For the isotropic case, the solution developed herein, reduces to the Donnell form of those given by Flugge.[9] Based on the generality of the solution procedure, the following effects can also be incorporated in the present

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